Seminar: Topological complexity: from critical points to robotics

- target: students of mathematics from Semester 5
- The seminar builds on some knowledge about homology and related issues. The contents of the course “Algebraic Topology I” are an ideal preparation.
- Time: Tue, 16:15-17:55 (to be rescheduled in case of conflicts: let me know)
- Ort: Sitzungszimmer
- Language: on agreement of all participants probably German, English is equally welcome
- Contact: Thomas Schick thomas.schick@math.uni-goettingen.de, Tel. 39-7799
- **Vorbesprechung Fr Feb 3, 13:15-, Sitzungsaal**

Mathematicians need to measure how “complicated” a space is.

A recent example: a “motion planner” for robots consists of a continuous choice of path for given initial and endpoint. Fact: only for very simple spaces (like $\mathbb{R}^2$) does this exist. For more complicated spaces, it can only be done on pieces of the space, and the smallest number of such pieces required to cover the space is one measure of topological complexity.

Higher dimensional spaces come up, if one has to solve the problem for several robots at the same time (without collisions).

Similarly, motivated by the calculus of variations in physices, given a manifold we’d like to know the minimum number of critical points any smooth function must have.

Defining such measures of complexity is one thing, computing them, or finding at least upper or lower bounds, a different one. To do this, many different tools from algebraic topology (homology, homotopy theory) can be used.

In the seminar, we will learn about several of these notions of complexity, compare them, develop algebraic topological tools to derive upper and lower bounds, and apply these bounds in many examples.

We will also delve a little bit in the applications from analysis and robotics alluded to.

The keywords are

- topology of the dynamics of the gradient flow of smooth functions
- topological robotics and motion planning
- Lusternik-Snirelmann category
- Topological ccomplexity of spaces
- cohomological cup-product length
- dimension theory for general topological spaces
- fibrations and lifting properties

Programm

This is a newly designed seminar. I hope the references are appropriate; the level is mixed, the starred ones are a bit more advanced.

Nr	Thema	Quelle
1	Ls-category and dimension. Intro, basic definitions, examples.	CLOT 1.2 without cohomology , Dim: xx
2	basics of cohomology; cup-length lower bound	CLOT 1.2 + (S) + xx (book on cohomology)
3^*	Dynamics of Morse and non-Morse gradient flows	CLOT 1.3 part, A.2, xx (Morse, OdE on Mf)
4^*	LS-Cat and number of critical points (using 3)	CLOT 1.3 rest
5	Motion planning and Topological complexity TC: intro	F1 1, F2 4, upto 4.8
6	TC and cohomology bounds	F1 4, F2 4.5 upto 4.43
7	Basic (homotopy) properties of LC-Cat and TC	CLOT 1.4, F1 2
8^*	Fibrations and lifting definitionso of LS-Cat I	CLOT 1.6 til 1.56, B.2, xx
9	TC with fibrations/dimension	F1 3, F2 4.2, rest
10	Simultaneous motion planning and its TC	F2 4.6
11	Collision free motion planning (config spaces and their TC)	F2 4.7
12^*	Motion planning in real projective space and immersions	FTY
13^*	Motion planning and complexity on graphs	F3 10-13 (selectin)
14^*	intro to homotopy groups; products in homotopy groups of group-like spaces and category	J 3

Literatur

(CLOT) Cornea, Lupton, Oprea, Tarre: Lusternik-Shnirelmann category. AMS, Providence 2003
(F1) Farber: Topological complexity of motion planning, Discrete Comput. Geom. 29 (2003), 211-221
(F2) Farber: Invitation to topological robotics. Zürich lectures in advanced mathematics. EMS, Zürich, 2008
(F3) Farber: Configuration spaces and robot motion planning algorithms; arXiv:1701.02083
(FTY) Farber, Tabachnikov, Yuzvinsky: Topological robotics: Motion planning in Projective spaces. arXiv:math/0210018
(J) James: On category, in the sense of Lusternik-Snirelmann. Topology 17 (1978) 331-348
(S) Schick: Quick and dirty intro to the cohomology ring. Unpublished manuscript, in stud.ip.
(xx) Quick introduction to topological dimension
(xx) Textbook on (co)homology
(xx) Introduction to Morse theory and OdE on manifolds
(xx) Introduction to fibrations and lifting properties