ALGEBRAIC AND COMBINATORIAL STRUCTURES IN KNOT THEORY

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Prerequisites for the course: Linear Algebra, Algebra
Instruction Language: English
Course Time: Summer Semester 2022 – Thursday 10.15-12.55 (?)
Pre-meeting date: April 11, 2022 @10 AM online

Description: Topology is a field of mathematics that studies geometrical objects by considering them made of rubber and thus instead of rigid measurements such as area, volume or angle, measurements subject to the rubber soul are the tools of topology. Topology roots back in the studies of the 19th century scientists such as Gauss, Tait, Ampere, Thomson. Gauss tried to understand earth' s magnetic potential via linked curves in space, Thomson suggested that atoms were knotted vortices in aether. These studies aroused a great mathematical interest in nicely shaped curves in space so called knots and with the developments in topology in the beginning of the 20th century, the study of knots became a mathematical theory on its own.

Knot theory is still one of the active areas in mathematics with many striking applications in biology (studies in DNA structure and enzymology), physics (quantum physics, Chern-Simons theory, Gauge theory), and chemistry (in molecule structure, synthesizing molecules). In this course, we will construct the fundamentals of knot theory, learn about mathematical tools for classifying knots, investigate the physical aspects of the theory and will discuss basic notions of algebraic topology and low-dimensional topology like homotopy, surfaces, 3-manifolds, and such.

Outline of the course: (subject to minor changes)

(1) Fundamentals of Knot Theory; equivalent definitions of a knot/link, homotopy, ambient isotopy, Reidemeister Theorem, Basic knot invariants, crossing number, stick number, bridge number
(2) Quandles, Biquandles
(3) Braids
(4) Braid group representations
(5) Jones polynomial
(6) Kauffman bracket
(7) Alexander polynomial

(8) Quantum models for knot polynomials
(9) Khovanov homology (may cover 2 weeks)
(10) Jones polynomial vs. Tutte polynomial

Reference Books

(1) Knots and Physics, Louis Kauffman
(2) Knot Theory, Charles Livingston
(3) Knots and Surfaces, N.D. Gilbert& T. Porter
(4) Knots and Links, D. Rolfsen