

RTG Lecture Course „Index Theorey and Theorems“ SoSe 25

- Target: PhD students of the RTG “Fourier Analysis and Spectral Theory”, master students with interest in analysis.
- Lecturer: Thomas Schick
- Area: Analysis of PdE, differential geometry, Geometric analysis, algebraic topology
- Time: Thu 14:15–15:55
- Place: Sitzungszimmer
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1 Course description

The index of a linear differential operator is the difference of the dimension of its kernel and its cokernel (provided finite). Its existence is known if the operator is elliptic on a compact manifold. The famous Atiyah-Singer index theorem gives a geometric formula for this index which does not depend on solving differential equations. The original proof uses K-theory and micro-local analysis. An alternative uses properties of the associated heat operator. Important modern generalizations cover more complicated situations (families, equivariance, non-compactness, not-quite-ellipticity).

The course will introduce the basic question, cover ideas of the original proofs and some of the modern developments.

In particular, we will present the so-called heat equation proof (which is more based on analysis) and the K-theoretic proof (which uses more topological principles); both with their own advantages and disadvantages (also in terms of generalizations).

NOTE: this course might have to be cancelled if the lecturer has to cover other teaching obligations for beginning students.