Rational Points on Hypersurfaces in Projective Space

Abstract

In this talk, we report on joint work with Andreas-Stephan Elsenhans. For the families $ax^3 = by^3+z^3+v^3+w^3,\ a,b = 1,\ \ldots\ ,100$, and $ax^4 = by^4 + z^4 + v^4 + w^4,\ a,b = 1,\ \ldots\ ,100$, of projective algebraic threefolds, we tested numerically the conjecture of Yu. I. Manin (in the refined form due to E. Peyre) about the asymptotics of points of bounded height on Fano varieties. This requires algorithms to solve Diophantine equations, to compute Peyre's Tamagawa-type number, and to detect accumulating subvarieties.

In this talk, we particularly emphasize the problem to search systematically for solutions of Diophantine equations. We describe an algorithm which was used to detect the only known non-obvious solution $(1\,484\,801, 1\,203\,120, 1\,169\,407, 1\,157\,520)$ of Sir P. Swinnerton-Dyer's equation $x^4 + 2y^4 = z^4 + 4w^4$.