

Construction of Rational Points on Rational Surfaces (Talk)

Abstract:

”Let K be a number field. Let S be a surface defined over K . We say that S is rational (over K) iff there is a birational map, defined over K , between S and K^2 . Obviously, rational surfaces have many points over K ; and conversely, for many rational surfaces it suffices to know one rational point in order to construct a rational parametrization (for instance in the case of quadric surfaces). Hence the problem of constructing rational points is closely related to constructing rational parametrizations.

In this lecture, we also discuss the problem of deciding rationality. Constructive Enriques-Manin reduction allows to reduce to minimal conic fibrations or to minimal Del Pezzo surfaces. For the case of fibrations, the problem is closely related to lattice reduction for quadratic forms over the ring $K[T]$ of univariate polynomials. The Del Pezzo case has a striking amount of symmetries, and we can exploit these using Lie algebras.”