EXERCISES RELATED TO THE TALK: CONSTRUCTION OF HYPERSURFACES WITH SINGULARITIES VIA FINITE FIELD EXPERIMENTS

OLIVER LABS

1. EXERCISES

You do not have to work through all the exercises from the first to the last one. Just choose those that you prefer.

(1) Construct (by hand!) a triangle-symmetric cubic surface with 4 ordinary real double points (also called nodes, locally $x^2 + y^2 + z^2 = 0$). For initial equations, see the sample script
`searchInFamilies_cub4_all.sin`
which can be downloaded from the workshop website.

(2) Use my visualization tool SURFEX (which is an extension of the raytracer SURF) to visualize the surface constructed in the previous exercise. It can be downloaded from the workshop website.

(3) Use elimination (and,if needed, primary decomposition) to construct a triangle-symmetric cubic surface with 4 nodes.

(4) Use the search over finite fields and lift technique to find a 4-gon-symmetric quartic surface with 16 nodes, e.g., using my (beta-versions of the) SINGULAR libraries
- `searchInFamilies.lib`
- `searchInFamilies_manySings.lib`
- `olitools.lib`

which can be downloaded from the workshop website. See also the script
`searchInFamilies_quart16_exercise.sin`
which can be downloaded from the workshop website.

(5) Use finite field experiments to guess geometric properties of a plane section of a 31-nodal 5-gon-symmetric quintic S, e.g., of $S|_E$ where E is one of the symmetry planes. See also the script
`searchInFamilies_quint31_exercise.sin`
which can be downloaded from the workshop website.

(6) Find another problem to which the search and lift technique can be applied and implement the corresponding function
`checkInterest(int chNo, ideal theI)`
for searching for an "interesting" algebraic variety and use the presented methods for finding it. Of course, such a function has to work both over finite prime fields and over characteristic zero.

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2. GETTING STARTED WITH SINGULAR

2.1. **Run and quit** SINGULAR**.** To start the SINGULAR program, type in a shell:

```
Singular
```

(or maybe `Singular-<Tab>` where `<Tab>` denotes the tabular key of your keyboard).

Then you can type in SINGULAR code. The first thing to do is usually to create a ring in which you want to work, with the characteristic, the variables, and the monomial ordering. A typical session could be:

```
LIB "all.lib";
ring r = 0, (x,y,z), dp;
poly f = x^3-y^2+z^2;
ideal j = f,jacob(f);
mult(std(j));
```

To quit SINGULAR, type:

```
quit;
```

or

```
$;
```

2.2. **Some sample Code.** Probably, the easiest way to get started is to adapt a sample script, e.g.:

```
searchInFamilies_cub4_all.sin
```

which can be downloaded from the workshop website.

To make this sample script work, you also have to download all the `.lib` files from the workshop website and place them into the same directory as the sample script.

2.3. **Running Existing Scripts.** Another way to work with SINGULAR is a non-interactive one; type:

```
Singular searchInFamilies_cub4_all.sin
```

This should run all the commands contained in the file and print some output.

2.4. **Visualization.** Finally, it tries to invoke my visualization tool SURFEX with a temporary file with a name similar to `surf7119.sux`. If you have not yet installed SURFEX this won't work. But if you have already done this then you will see some windows showing up one of which shows the 4-nodal cubic surface which was constructed using the SINGULAR script a few seconds before. You can rotate the (yellow) surface by dragging the (green) ball in the other window.

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